Microwave Propagation in Semiconductors with Carrier Density Varying in Time*

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Summary—Expressions are derived for the propagation function of a microwave signal inside a waveguide filled with a semiconductor with carrier density varying in time. The bearing of these expressions on the performance of semiconductor microwave modulators and on microwave methods for the measurement of lifetime in semiconductors is discussed.

Introduction

MICROWAVE modulators using semiconductors have been proposed by Jacobs, *et al.*[1] The main component of the modulator is a semiconductor sample mounted in a waveguide. Modulation is introduced by injecting carriers at the modulation frequency. Microwave power absorbed in the sample varies due to the increased carrier concentration and the total power transmitted through the semiconductor is thus modulated.

The microwave absorption has also been used to measure the minority carrier lifetime by measuring the decay time of the transmitted power on rapidly withdrawing the injecting source.[2,3]

The purpose of this paper is to make a theoretical study of the propagation of a microwave signal under the above-mentioned experimental conditions. Expressions for the electric field intensity in the waveguide are obtained and the results are discussed in relation to the above applications.

The Field Equations

A TE_{01} mode wave is assumed to propagate in a guide filled with a semiconductor sample having the geometry shown in Fig. 1. The equation giving the electric field intensity, E_x, in the guide can be written as

$$\frac{1}{\mu}\left[\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}\right]=\frac{\partial^2}{\partial t^2}(\epsilon E_x)+\frac{\partial}{\partial t}(\sigma E_x) \tag{1}$$

where σ, ϵ, and μ are, respectively, the conductivity, permittivity and permeability of the semiconductor.

The conductivity and also the permittivity is affected at the microwave frequencies by the carrier density.[4,5,6] In general, when the carrier density is modulated both σ and ϵ change with time. The change in permittivity for a change in carrier concentration is, however, dependent on the initial carrier concentration and other parameters of the sample.

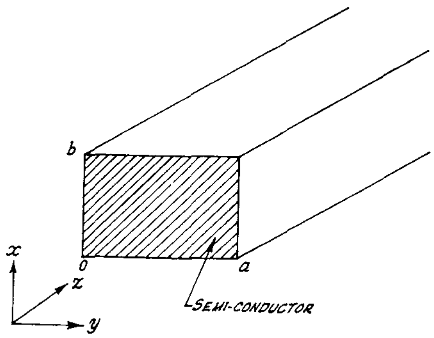

Fig. 1—The coordinate axes for the waveguide.

Eq. (1) can be written as

$$\frac{1}{\mu}\left[\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}\right]=\epsilon\frac{\partial^2 E_x}{\partial t^2}+2\frac{\partial\epsilon}{\partial t}\frac{\partial E_x}{\partial t}+E_x\frac{\partial^2\epsilon}{\partial t^2}+\sigma\frac{\partial E_x}{\partial t}+E_x\frac{\partial\sigma}{\partial t}\cdot \tag{2}$$

Now, in the arrangement for measuring lifetime one introduces carriers by the injection of light or through point contacts. After the steady state is reached, the injection is quickly stopped and the extra carriers decay exponentially. In the following analysis it is assumed that the injection is homogeneous and, also, that the effect of surface recombination is neglected.

The carrier density at any instant after the injection is stopped is given by

$$N = N_0 + \Delta N e^{-t/\tau}$$

where N_0 is the equilibrium carrier density with no injection, $N_0+\Delta N$ is the carrier density at the instant of stopping the injection, and τ is the lifetime.

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[1] H. Jacobs, F. A. Brand, M. Benanti, R. Benjamin and J. Meindl, "A new semiconductor microwave modulatior," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 553–559; September, 1960.

[2] A. P. Ramsa, H. Jacobs and F. A. Brand, "Microwave techniques in measurement of lifetime in germanium," *J. Appl. Phys.*, voo. 30, pp. 1054–1060; July, 1959.

[3] R. D. Larrabee, "Measurement of semiconductor properties through microwave absorption," *RCA Rev.*, vol. 21, pp. 124–129; March, 1960.

[4] T. S. Benedict and W. Shockley, "Microwave observation of the collision frequency of electrons in germanium," *Phys. Rev.*, vol. 89, pp. 1152–1153; March, 1953.

[5] T. S. Benedict, "Microwave observation of the collision frequency of holes in germanium," *Phys. Rev.*, vol. 91, pp. 1565–1566; September, 1953.

[6] H. A. Atwater, "Microwave measurement of semiconductor carrier lifetime," *J. Appl. Phys.*, vol. 31, p. 938; May, 1960.

The conductivity σ and the permittivity ϵ may be taken to be

$$\sigma = \sigma_s + \sigma_1 e^{-t/\tau}$$
$$\epsilon = \epsilon_s - \epsilon_1 e^{-t/\tau} \tag{3}$$

where σ_s, ϵ_s are the unperturbed values of σ and ϵ. The perturbations are given by σ_1 and ϵ_1. It is to be noted that the change in σ is, in general, of opposite sign to the change in ϵ.

For the modulator if the carrier density is varied sinusoidally one has similarly

$$\sigma = \sigma_s + \sigma_1 \cos \omega_m t$$
$$\epsilon = \epsilon_s - \epsilon_1 \cos \omega_m t \tag{4}$$

where ω_m is the modulation frequency. Eq. (2) will be solved, when σ and ϵ are given by (3). The results enable one to write directly the solutions for the modulator.

Substituting for σ and ϵ from (3) in (2),

$$\frac{1}{\mu}\left[\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right] = (\epsilon_s - \epsilon_1 e^{-t/\tau})\frac{\partial^2 E_x}{\partial t^2} + \left[\sigma_s + \left(\sigma_1 + \frac{2\epsilon_1}{\tau}\right)e^{-t/\tau}\right]\frac{\partial E_x}{\partial t} - \frac{1}{\tau}\left(\frac{\epsilon_1}{\tau} + \sigma_1\right)e^{-t/\tau}E_x. \tag{5}$$

Expression for the Electric Field

The electric field intensity, E_n, given by (5) may be written as

$$E_x = E_{x0} e^{-j(\gamma_s z - \omega t) + \beta(z,t)} \tag{6}$$

where $\gamma_s = [\omega^2 \mu \epsilon_s - (\pi/a)^2 - j\omega\mu\sigma_s]^{1/2}$ is the propagation function for the unchanged semiconductor. The term $\beta(z, t)$ is the perturbation in the value of propagation function introduced by the change in dielectric constant ϵ_1 and in conductivity σ_1.

The perturbation term $\beta(z, t)$ satisfies the equation

$$\frac{1}{\mu}\frac{\partial^2 \beta}{\partial z^2} + \frac{1}{\mu}\left(\frac{\partial \beta}{\partial z}\right)^2 - 2j\frac{\gamma_s}{\mu}\frac{\partial \beta}{\partial z} - \frac{\partial^2 \beta}{\partial t^2}(\epsilon_s - \epsilon_1 e^{-t/\tau}) - \left(\frac{\partial \beta}{\partial t}\right)^2(\epsilon_s - \epsilon_1 e^{-t/\tau}) - \frac{\partial \beta}{\partial t}\left[\sigma_s + 2j\omega\epsilon_s + \left(\sigma_1 + \frac{2\epsilon_1}{\tau} - 2j\omega\epsilon_1\right)e^{-t/\tau}\right] = \left[\left(\omega^2\epsilon_1 - \frac{\epsilon_1}{\tau^2} - \frac{\sigma_1}{\tau}\right) + j\omega\left(\sigma_1 + \frac{2\epsilon_1}{\tau}\right)\right]e^{-t/\tau}. \tag{7}$$

Evidently β would be of the order of

$$\gamma_s\left(\frac{\epsilon_1}{\epsilon_s} + j\frac{\sigma_1}{\omega\epsilon_s}\right)$$

and since ϵ_1/ϵ_s and $\sigma_1/\omega\epsilon_s$ are assumed to be small, a solution of (7) may be obtained in a series of the form

$$\beta = \gamma_s(\beta_1 + \beta_2 + \cdots) \tag{8}$$

where β_1 is of the order of

$$\left(\frac{\epsilon_1}{\epsilon_s} + j\frac{\sigma_1}{\omega\epsilon_s}\right),$$

β_2 is the order of

$$\left(\frac{\epsilon_1}{\epsilon_s} + j\frac{\sigma_1}{\omega\epsilon_s}\right)^2$$

and so on.

The equation giving β_1 is obtained by replacing β in (7) by $\gamma_s\beta_1$, neglecting the second derivatives of β and the products of the first derivatives of β and ϵ_1, σ_1. One then obtains

$$\frac{\partial \beta_1}{\partial z} - \alpha_1\frac{\partial \beta_1}{\partial t} = \alpha_2 e^{-t/\tau} \tag{9}$$

where

$$\alpha_1 = -\frac{\mu}{2j\gamma_s}(\sigma_s + 2j\omega\epsilon_s) \tag{10}$$

$$\alpha_2 = -\frac{\mu}{2j\gamma_s^2}\left[\left(\omega^2\epsilon_1 - \frac{\epsilon_1}{\tau^2} - \frac{\sigma_1}{\tau}\right) + j\omega\left(\sigma_1 + \frac{2\epsilon_1}{\omega\tau}\right)\right]. \tag{11}$$

The first term of the series β_1 is hence given by

$$\beta_1 = \frac{\alpha_2\tau}{\alpha_1}\left[e^{-t/\tau} - e^{-1/\tau(\alpha_1 z + t)}\right]. \tag{12}$$

Eq. (12) can be simplified considering that

$$\frac{\alpha_1 z}{\sigma} \approx \frac{\sqrt{\mu\epsilon_s}}{\tau}z = \frac{2\pi}{\omega\tau}\cdot\frac{z}{\lambda_s} \tag{13}$$

where λ_s is the wavelength in the free semiconductor.

Hence, if the modulation rate is small compared to the microwave frequency as is really the case for either the modulator or the lifetime experiment the quantity $\alpha_1 z/\tau$ can be considered to be much less than one. One may then write β_1 as

$$\beta_1 = \alpha_2 z e^{-t/\tau}. \tag{14}$$

The equation giving the second term of the series β_2 is obtained by replacing β in (7) by $\gamma_s(\beta_1+\beta_2)$ and retaining only the terms of the order of α_2^2. One obtains

$$\frac{\partial \beta_2}{\partial z} - \alpha_1\frac{\partial \beta_2}{\partial t} = \frac{\alpha_2^2}{2j}e^{-2/\tau(\alpha_1 z + t)}. \tag{15}$$

The solution of (15) when approximation (13) is used may be written as

$$\beta_2 = \frac{\alpha_2^2 \tau}{4j\alpha_1}\left[e^{-2t/\tau} - e^{-2/\tau(\alpha_1 z + t)}\right] \approx \frac{\alpha_2^2}{2j} z e^{-2t/\tau}. \tag{16}$$

The higher-order terms in the solution may be obtained on repeating the above procedure. However, it is noted that so long as (13) is valid the higher-order solutions include only the terms having higher periodicity. There magnitude is α_2 times less than that of the fundamental. The effect of the higher-order terms may therefore be neglected if the change in carrier density is kept low. The only requirement for this approximation to be correct is that $\alpha_1 z/\tau$ should be small. It may be pointed out that σ_s need not be small compared to ωt_s as has been suggested by some authors.

In the following will be discussed the general features of propagation that one obtains on considering the first-order term only.

Discussion

The electric field intensity is given correct to first order by

$$E_x = E_{x0} e^{-j(\gamma_s z - \omega t)} e^{mze^{-t/\tau} + jnze^{-t/\tau}} \tag{17}$$

where $m + jn = \gamma_s \alpha_2$. The effective change in propagation constant is $(m + jn)e^{-t/\tau}$. It is evident from (17) that due to modulation of the carrier density both the phase and amplitude of the microwave signal is modulated. The effect of the change in dielectric constant is similar to that of the change in conductivity.

The phase modulation is due to $nze^{-t/\tau}$. It is proportional to z and also the wave shape is the same as that of the modulating signal.

The amplitude factor is

$$e^{-(\alpha_s - me^{-t/\tau})z} \tag{18}$$

where α_s is the imaginary part of γ_s. The power transmitted through a length d of the sample is

$$P_t = e^{-2(\alpha_s - me^{-t/\tau})d} P_{\text{in}} \tag{19}$$

where P_{in} is the input power.

The transmitted power reproduces the shape of modulation only if $\alpha_s d$ is a small quantity so that (P_t/P_{in}) may be approximated to

$$\frac{P_t}{P_{\text{in}}} = 1 - 2(\alpha_s - me^{-t/\tau})d. \tag{20}$$

It may be pointed out that if instead of displaying P_t one displays the logarithm of P_t, the decay of P_t would give the lifetime correctly since

$$\ln P_t = -2\alpha_s d + \ln P_{\text{in}} + 2mde^{-t/\tau}. \tag{21}$$

This is evidently true even if $\alpha_s d$ is not a small quantity.

The above discussion leads to the conclusion that the microwave lifetime method gives τ correctly if $\ln P_t$ is displayed, and for the linear display if the attenuation is small. The restriction $(\sigma_s/\omega t_s) \ll 1$ is eliminated for the logarithmic display. On the other hand, if the phase of the signal is displayed, the modulation of phase would give τ correctly. The initial change in carrier density should, however, be kept small.

For sinusoidal modulation one obtains for the amplitude factor

$$e^{-(\alpha_s - m\cos\omega_m t)d} \tag{22}$$

and for the phase factor

$$nd \cos \omega_m t. \tag{23}$$

Thus, the amplitude modulation envelope is sinusoidal only if $\alpha_s d \ll 1$ or, in other words, the depth of modulation is small. The phase modulation is, however, sinusoidal even for large values of $\alpha_s d$. If a low loss material, in which modulation of σ or ϵ may be produced by injection of carriers, is available, large depth of phase modulation may be achieved by using suitable length of the material.

Reflection from a Semi-Infinite Sample

In the previous discussion the semiconductor was assumed to fill all the guide. However, the effect of reflection at one surface of the semiconductor sample partially filling the guide may be obtained using the expression for the propagation constant obtained earlier.

The characteristic impedance of the semiconductor-filled guide section is

$$Z_s = \frac{\omega\mu}{\gamma_s(1 + j\alpha_2 e^{-t/\tau})}\cdot \tag{24}$$

Using the above value for Z_s, general expressions for the power received at the detector terminal in the arrangement of Fig. 2 may be obtained by using the general transmission line theory. In general, these expressions will be rather complex.

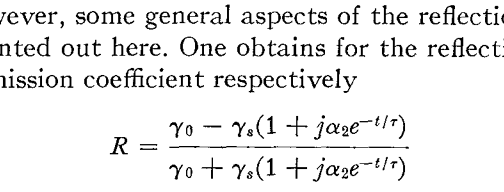

Fig. 2—Experimental arrangement for the detection of power transmitted through a semi-infinite sample.

However, some general aspects of the reflection may be pointed out here. One obtains for the reflection and transmission coefficient respectively

$$R = \frac{\gamma_0 - \gamma_s(1 + j\alpha_2 e^{-t/\tau})}{\gamma_0 + \gamma_s(1 + j\alpha_2 e^{-t/\tau})} \tag{25}$$

$$T = \frac{2\gamma_0}{\gamma_0 + \gamma_s(1 + j\alpha_2 e^{-t/\tau})} \tag{26}$$

where γ_0 is the propagation constant for the free guide. Eq. (25) may be simplified to

$$R = R_0\left[\frac{1 - (j\gamma_s\alpha_2 e^{-t/\tau})/(\gamma_0 - \gamma_s)}{1 + (j\gamma_s\alpha_2 e^{-t/\tau})/(\gamma_0 + \gamma_s)}\right] \tag{27}$$

where R_0 is the reflection coefficient before modulation.

The first-order approximation of R is

$$R = R_0[1 - (2j\gamma_0\gamma_s\alpha_2 e^{-t/\tau})/(\gamma_0^2 - \gamma_s^2)]. \tag{28}$$

Evidently then the information relating to lifetime may also be obtained by observing the reflected signal only.[7] It should, however, be noted that due to the increased carrier density both the magnitude and phase of reflection is changed. Hence, the change in the magnitude of the reflected signal will have the wave shape of the modulation only if it is observed alone. If the detected signal is due to the combination of both the incident and reflected signal, the wave shape may be very different.

On the other hand, if the transmitted signal is observed by locating a probe inside the semiconductor or by a matched detector, the observations made in the previous section are mostly applicable, only the amplitude is multiplied by the factor T. The effect of the change in carrier density on the value of T may be considered to be small. It will be more so in the logarithmic display, as suggested earlier.

[7] S. Deb and B. R. Nag, "Measurement of lifetime of carriers in semiconductors through microwave reflection," *J. Appl. Phys.*, vol. 33, p. 1604; April, 1962.

Conclusion

Expressions have been obtained for electromagnetic fields propagating in a waveguide filled with a semiconductor with carrier density varying with time. It has been shown that due to the variation of carrier density both the amplitude and phase modulation occur. The phase modulation reproduces the wave shape correctly. The wave shape of amplitude modulation is correct only for small attenuation. However, the logarithm of the amplitude reproduces the wave shape correctly even for larger attenuation.

It has also been shown that the reflected signal from a semiconductor surface may be used to measure the lifetime. The effect of reflection on the wave shape of the transmitted signal is also found to be small.

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On the Resonant-Cavity Method for Measurement of Varactors*

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Summary—**An extension of Houlding's method for measurement of varactor quality has been developed by using a swept-bias voltage instead of a discrete step in bias. The method can be used to measure the cutoff frequency of high-Q varactors if the law of capacitance variation is known. In this case, it is not necessary to measure capacity or reactance since cutoff frequency is shown to be determined by reflected power on a microwave-reflectometer system. The Sweep-Voltage Method places emphasis on the measurement of reflected power and a test bench is described for application of the method.**

The series resistance of high-voltage epitaxial varactors is shown, by calculation and experimental data, to be a function of bias. This is due to space-charge layer widening which causes the epitaxial base layer to vary in length. For this reason it is advantageous to compare high-voltage epitaxial varactors by sweeping the bias over a broad range of the diode characteristics. Data is given for typical per cent of reflected microwave power $\tilde{R}$ for 6.5-, 15-, and 100-v varactors where the varactor is matched to the line at 2.00-, 5.00- and 15.0-v bias and a sine wave at 100 kc with amplitude 4.00, 10.0 or 30.0 v is superimposed on the bias. The measurements are independent of the power incident on the varactor to a level of 700 μw.

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I. Introduction

THE QUALITY of a microwave varactor diode is related to the losses in and near the p-n junction which provides the voltage-variable reactance. These losses are accepted to be equivalent to a constant resistance R_s in series with a voltage-variable capacitance. A method for measurement of R_s has been described by Houlding[1] wherein the varactor is in a

[1] N. Houlding, "Measurement of varactor quality," *The Microwave J.*, p. 40; January, 1960.